Lower bound for sum of medians in non-obtuse triangle.
https://www.linkedin.com/feed/update/urn:li:activity:6489817599789146112
Prove that if a triangle is not obtuse, then the sum of the lengths of its medians is at least four times the circumradius of the triangle.

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Since* $m_a \geq \dfrac{b^2+c^2}{4R}$, $m_b \geq \dfrac{c^2+a^2}{4R}$, $m_c \geq \dfrac{a^2+b^2}{4R}$ and in non-obtuse triangle holds inequality* $a^2+b^2+c^2 \geq 8R^2$ then $m_a+m_b+m_c = \dfrac{a^2+b^2+c^2}{4R} \geq \dfrac{8R^2}{4R} = 4R$.

* Let R and d_a be, respectively, circumradius and distance from the circumcenter to side a. Then by triangle inequality $|m_a - R| \leq d_a$ and, since $d_a = \sqrt{R^2 - \dfrac{a^2}{4}}$ then we obtain:

$|m_a - R| \leq \sqrt{R^2 - \dfrac{a^2}{4}} \Leftrightarrow m_a^2 - 2m_aR + R^2 \leq R^2 - \dfrac{a^2}{4} \Leftrightarrow 4m_a^2 - 8m_aR + a^2 \leq 0 \Leftrightarrow$
$2(b^2+c^2) - a^2 - 8m_aR + a^2 \leq 0 \Leftrightarrow b^2+c^2 \leq 4m_aR$.

** *Since* $8R^2 - (a^2+b^2+c^2) = 2R^2(4 - 2\sin^2 A - 2\sin^2 B - 2\sin^2 C) =$
$2R^2(1 + \cos 2A + \cos 2B + \cos 2B) = -8R^2 \cos\alpha \cos\beta \cos\gamma$ *then triangle is acute-angled, right-angled or obtuse-angled iff* $a^2+b^2+c^2 > 8R^2$, $a^2+b^2+c^2 = 8R^2$ *or* $a^2+b^2+c^2 < 8R^2$ *respectively.*